

# 1 Bayes Theorem and Independence

## 1.1 Concepts

1. We use **Bayes theorem** when we want to find the probability of  $A$  given  $B$  but we are told the opposite probability, the probability of  $B$  given  $A$ . There are several forms of Bayes Theorem as follows:

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)} = \frac{P(B|A)P(A)}{P(B|A)P(A) + P(B|\bar{A})P(\bar{A})} = \frac{1}{1 + \frac{P(B|\bar{A})P(\bar{A})}{P(B|A)P(A)}}.$$

In order to discern which form to use, look at the information you are given. If you are told  $P(B|A)$  as well as  $P(B|\bar{A})$ , use the latter two methods but if you are only told  $P(B)$ , then use the first form.

We say that two events  $A, B$  are **independent** if  $P(A \cap B) = P(A)P(B)$ .

## 1.2 Examples

2. When rolling a fair 6-sided die, are the events  $A$  that the number rolled is greater than or equal to 3, and  $B$  that the number rolled is odd, independent?
3. There are 10 red and 10 blue balls in a bag. Someone randomly picks out a ball and then places it back and puts 10 more balls of that color into the bag. Then you draw a ball. What is the probability that the 10 balls added were red, given that you drew out a red ball?

## 1.3 Problems

4. True    False    If  $A, B$  are mutual exclusive events that are independent, then  $P(A) = 0$  or  $P(B) = 0$ .
5. True    False    If  $A, B$  are independent events and  $B, C$  are independent, then  $A, C$  are independent.
6. I roll two die. Are the events that the first die roll is a 1 and that the sum of the two dice is a 7 independent?
7. What is the probability that a family with two kids has two boys if you know at least one is a boy?